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# Influence of intrinsic decoherence on nonclassical properties of the two-mode Raman coupled model

J.-B. Xu[a], X.-B. Zou, and J.-H. Yu

Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, P.R. China

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**Abstract.** The two-mode Raman coupled model governed by the Milburn equation is studied by the dynamical algebraic method. With the help of an $SU(2)$ dynamical algebraic structure, we find an exact solution of the Milburn equation for an effective two-mode Raman coupled Hamiltonian. The exact solution is then used to discuss the influence of intrinsic decoherence on nonclassical properties of the system, such as collapses and revivals of the atomic inversion, oscillations of the photon number distribution and squeezing of the radiation field.

**PACS.** 42.50.-p Quantum optics – 32.80.-t Photon interactions with atoms – 02.20.-a Group theory

## 1 Introduction

In the past two decades, there has been great interest in the Jaynes-Cummings model [1] of a single two-level atom interacting with a single mode of a quantized cavity field. The Jaynes-Cummings model may be the simplest solvable model describing the interaction between two dissimilar quantum systems and exhibits many interesting quantum effects, such as collapses and revivals of Rabi oscillations [2,3], squeezing of the radiation field [4], sub-Poissonian statistics [5], etc. The development of the experiments in high-$Q$ superconductivity cavities has demonstrated these interesting nonclassical properties and simulated the interest of studying the Jaynes-Cummings model and its various generalizations.

In recent years, much attention has been focused on the study of an effective two-level atom interacting with two quantized field modes through a Raman interaction, called two-mode Raman coupled model [6–8]. It has also been noted that the dynamics of such a system reveals many remarkable features that are very different from those of the usual Jaynes-Cummings model. The main advantage of this model is that one can use one mode to modulate or control the output of the other mode.

On the other hand, there has been increased interest in the problem of decoherence in quantum mechanics because of its possible applications in quantum measurement processes and quantum computers [9]. In recent years, there have several proposals to solve the decoherence problem. In particular, Milburn [10] has proposed a simple model of intrinsic decoherence based on an assumption that on sufficiently short time steps the system does not evolves continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations. This model gives a simple modification of standard quantum mechanics and the quantum coherence is automatically destroyed as the quantum system evolves. Milburn considered only the free evolution of a simple quantum system. The intrinsic decoherence in the Jaynes-Cummings model with one quantized field mode has been studied [11,12].

In this paper, we will consider an effective two-mode Raman coupled model governed by the Milburn equation. Based on an $SU(2)$ dynamical algebraic structure, we find an exact solution of the Milburn equation for the effective two-mode Raman coupled Hamiltonian and apply it to study the influence of intrinsic decoherence on nonclassical properties of the system. It is shown that the intrinsic decoherence in the atom-field interaction suppresses these nonclassical effects in the effective two-mode Raman coupled model.

The paper is organized as follows. In Section 2, we present an exact solution of the Milburn equation for the effective two-mode Raman coupled model by making use of dynamical algebraic method. In Section 3, we study the influence of intrinsic decoherence on nonclassical properties of the system, such as collapses and revivals of the atomic inversion, oscillations of the photon number distribution and squeezing of the radiation field. In Section 4, there are some concluding remarks.

## 2 Exact solution of the Milburn equation

We consider a quantum system described by the density operator $\rho(t)$. In standard quantum mechanics, dynamics of the system is governed by the evolution operator $U(t) = \exp(-\mathrm{i}Ht)$, where $H$ is Hamiltonian of the system. Milburn assumed [10] that on sufficiently short time

[a] e-mail: phy_xujb@dial.zju.edu.cn

steps the system does not evolves continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations. Based on this assumption, he has derived the equation for the time evolution density operator $\rho(t)$ of the quantum system [10]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \gamma\left[\exp\left(-\frac{\mathrm{i}}{\gamma}H\right)\rho(t)\exp\left(\frac{\mathrm{i}}{\gamma}H\right) - \rho(t)\right], \quad (1)$$

where $\gamma$ is the mean frequency of the unitary time step. This equation formally corresponds to the assumption that on sufficiently short time steps the system evolves with a probability $p(\tau) = \gamma\tau$. In the limit $\gamma \longrightarrow \infty$, equation (1) reduces to the ordinary von Neuman equation for the density operator.

Expanding equation (1) to first order in $\gamma^{-1}$, Milburn obtained the following dynamical equation,

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}[H, \rho(t)] - \frac{1}{2\gamma}[H, [H, \rho(t)]], \quad (2)$$

which is the Milburn equation. Milburn discussed the solution of equation (2) for a harmonic oscillator and a processing spin system. The exact solution of the Milburn equation for Jaynes-Cumming model with one quantized mode has also been obtained [11,12]. In this paper, we will study an effective two-mode Raman coupled model governed by the Milburn equation and present an exact solution of Milburn equation for the system.

We introduce three auxiliary superoperators $R, S, T$ defined by

$$\begin{aligned}
&\exp(R\tau)\rho(t) = \sum_{k=0}^{\infty}\frac{1}{k!}H^k\rho(t)H^k;\\
&\exp(S\tau)\rho(t) = \exp(-\mathrm{i}H\tau)\rho(t)\exp(\mathrm{i}H\tau);\\
&\exp(T\tau)\rho(t) = \exp\left(-\frac{\tau}{2\gamma}H^2\right)\rho(t)\exp\left(-\frac{\tau}{2\gamma}H^2\right), \quad (3)
\end{aligned}$$

which lead to

$$R\rho = \frac{1}{\gamma}H\rho H;\quad S\rho = -\mathrm{i}[H, \rho];\quad T\rho = \frac{1}{2\gamma}\{H^2, \rho\}, \quad (4)$$

where $H$ is the Hamiltonian of the system. From equations (3, 4), it is easy to obtain the formal solution of the Milburn equation as follows

$$\begin{aligned}
\rho(t) &= \mathrm{e}^{Rt}\mathrm{e}^{St}\mathrm{e}^{Tt}\rho(0)\\
&= \sum_{k=0}^{\infty}(\frac{t}{\gamma})^k\frac{1}{k!}H^k\mathrm{e}^{-\mathrm{i}Ht}\mathrm{e}^{-\frac{1}{2\gamma}H^2t}\rho(0)\mathrm{e}^{-\frac{1}{2\gamma}H^2t}\mathrm{e}^{\mathrm{i}Ht}H^k\\
&= \sum_{k=0}^{\infty}(\frac{t}{\gamma})^k\frac{1}{k!}M^k\rho(0)M^{\dagger k}, \quad (5)
\end{aligned}$$

where $\rho(0)$ is the density operator of the initial atom-field system and $M^k$ is defined by

$$M^k = H^k\exp(-\mathrm{i}Ht)\exp\left(-\frac{t}{2\gamma}H^2\right). \quad (6)$$

In the following, we will solve the Milburn equation for the two-mode Raman coupled model with the Hamiltonian [6]

$$H = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \frac{\omega}{2}\sigma_z + g(a_1 a_2^\dagger\sigma_+ + a_1^\dagger a_2\sigma_-), \quad (7)$$

where $\sigma_z$, $\sigma_\pm$ are the atomic spin flip operators characterizing the effective two-level atom with transition frequency $\omega$ and $a_1(a_2)$, $a_1^\dagger(a_2^\dagger)$ are annihilation (creation) operators of the 1st (2nd) mode light field of frequencies $\omega_1(\omega_2)$ respectively. The Hamiltonian (7) ignores stark shifts and the parameter $g$ is the atom-field coupling constants.

It is easily proved that there exists two constants of motion in the Hamiltonian (7)

$$K_1 = a_1^\dagger a_1 + \frac{1+\sigma_z}{2};\quad K_2 = a_2^\dagger a_2 + \frac{1-\sigma_z}{2}, \quad (8)$$

which commute with Hamiltonian (7). We then define the operators

$$S_0 = \frac{\sigma_z}{2};\ S_+ = \frac{a_1 a_2^\dagger\sigma_+}{\sqrt{K_1K_2}};\ S_- = \frac{a_1^\dagger a_2\sigma_-}{\sqrt{K_1K_2}}, \quad (9)$$

we can show that operators $S_\pm, S_0$ satisfy the following commutation relations

$$[S_0, S_\pm] = \pm S_\pm;\ [S_+, S_-] = 2S_0, \quad (10)$$

which constitute an $SU(2)$ algebra. In term of the $SU(2)$ generators, we can rewrite the Hamiltonian (7) as

$$\begin{aligned}
H = &\ \omega_1\left(K_1 - \frac{1}{2}\right) + \omega_2\left(K_2 - \frac{1}{2}\right) + \Delta S_0\\
&+ g\sqrt{K_1K_2}(S_+ + S_-), \quad (11)
\end{aligned}$$

where $\Delta = \omega + \omega_2 - \omega_1$. With the help of the $SU(2)$ dynamical algebraic structure, we can diagonalize the Hamiltonian (11) by unitary transformations,

$$U = \exp\left[\frac{\theta(K_1, K_2)}{2}(S_+ - S_-)\right], \quad (12)$$

with

$$\theta(K_1, K_2) = \tan^{-1}\frac{2g\sqrt{K_1K_2}}{\Delta}, \quad (13)$$

and get transformed Hamiltonian

$$\begin{aligned}
H' &= UHU^\dagger\\
&= \omega_1\left(K_1 - \frac{1}{2}\right) + \omega_2\left(K_2 - \frac{1}{2}\right) + 2\Omega(K_1K_2)S_0, \quad (14)
\end{aligned}$$

where

$$\Omega(K_1, K_2) = \sqrt{\frac{\Delta^2}{4} + g^2K_1K_2}. \quad (15)$$

$$\rho^n(t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty} Q_{n_1}Q_{n_2}Q^*_{m_1}Q^*_{m_2}|\psi^n(e,n_1,n_2)\rangle\langle\psi^n(e,m_1,m_2)|; \quad (22)$$

$$\begin{aligned}|\psi^n(e,n_1,n_2)\rangle &= M^n|e,n_1,n_2\rangle \\ &= \frac{1}{2}\left\{f_+^n(n_1+1,n_2)\exp[-\mathrm{i}f_+(n_1+1,n_2)t]\exp\left[-\frac{f_+^2(n_1+1,n_2)t}{2\gamma}\right]\right. \\ &\quad \left. - f_-^n(n_1+1,n_2)\exp[-\mathrm{i}f_-(n_1+1,n_2)t]\exp\left[-\frac{f_-^2(n_1+1,n_2)t}{2\gamma}\right]\right\}\frac{g\sqrt{(n_1+1)n_2}}{\Omega(n_1+1,n_2)}|n_1+1,n_2-1,g\rangle \\ &\quad + \frac{1}{2}\left\{f_+^n(n_1+1,n_2)\exp[-\mathrm{i}f_+(n_1+1,n_2)t]\exp\left[-\frac{f_+^2(n_1+1,n_2)t}{2\gamma}\right]\left[1+\frac{\Delta}{2\Omega(n_1+1,n_2)}\right]\right. \\ &\quad \left. + f_-^n(n_1+1,n_2)\exp[-\mathrm{i}f_-(n_1+1,n_2)t]\exp\left[-\frac{f_-^2(n_1+1,n_2)t}{2\gamma}\right]\left[1-\frac{\Delta}{2\Omega(n_1+1,n_2)}\right]\right\}|n_1,n_2,e\rangle; \quad (23)\end{aligned}$$

$$\begin{aligned}f_\pm(n_1+1,n_2) &= \omega_1\left(n_1+\frac{1}{2}\right)+\omega_2\left(n_2-\frac{1}{2}\right)\pm\Omega(n_1+1,n_2); \\ \Omega(n_1+1,n_2) &= \sqrt{\frac{\Delta^2}{4}+g^2(n_1+1)n_2} \quad (24)\end{aligned}$$

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In terms of equations (12, 14), it is easy to obtain

$$\begin{aligned}M^k &= U^\dagger H'^k\exp(-\mathrm{i}H't)\exp\left(-\frac{t}{2\gamma}H'^2\right)U \\ &= \frac{1}{2}\left[\hat{f}_+^k\exp(-\mathrm{i}\hat{f}_+t)\exp\left(-\frac{\hat{f}_+^2t}{2\gamma}\right)\right. \\ &\quad \left. +\hat{f}_-^k\exp(-\mathrm{i}\hat{f}_-t)\exp\left(-\frac{\hat{f}_-^2t}{2\gamma}\right)\right] \\ &\quad + \frac{1}{2}\left[\hat{f}_+^k\exp(-\mathrm{i}\hat{f}_+t)\exp\left(-\frac{\hat{f}_+^2t}{2\gamma}\right)\right. \\ &\quad \left. -\hat{f}_-^k\exp(-\mathrm{i}\hat{f}_-t)\exp\left(-\frac{\hat{f}_-^2t}{2\gamma}\right)\right] \\ &\quad \times\left[\frac{\Delta\sigma_z}{2\Omega(K_1,K_2)}+\frac{g(a_1a_2^\dagger\sigma_+ + a_1^\dagger a_2\sigma_-)}{\Omega(K_1,K_2)}\right], \quad (16)\end{aligned}$$

where

$$\hat{f}_\pm = \omega_1\left(K_1-\frac{1}{2}\right)+\omega_2\left(K_2-\frac{1}{2}\right)\pm\Omega(K_1,K_2). \quad (17)$$

We now assume that initially the fields are prepared in coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$:

$$|\psi(0)\rangle_F = |\alpha_1\rangle|\alpha_2\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}Q_{n_1}Q_{n_2}|n_1,n_2\rangle, \quad (18)$$

where

$$Q_{n_i} = \mathrm{e}^{-\frac{1}{2}|\alpha_i|^2}\frac{\alpha_i^{n_i}}{\sqrt{n_i!}}, \quad (19)$$

and the atom was prepared in excited states $|e\rangle$. Then the density operator $\rho(0)$ of the initial state can be written as

$$\begin{aligned}\rho(0) = &\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}Q_{n_1}Q_{n_2}Q^*_{m_1}Q^*_{m_2} \\ &\times|e,n_1,n_2\rangle\langle e,m_1,m_2|. \quad (20)\end{aligned}$$

Substituting equations (16, 20) into (5), we can obtain explicit expression for the density operator $\rho(t)$ as follows:

$$\rho(t) = \sum_{n=0}^{\infty}\left(\frac{t}{\gamma}\right)^n\frac{1}{n!}\rho^n(t), \quad (21)$$

where

*see equations (22–24) above.*

Equations (21–24) represent the exact solution of the Milburn equation for the two-mode Raman coupled model with Hamiltonian (7). The advantage of our approach is that it is easy to study the dynamics and statistics of atom and field quantity for arbitrary initial states.

## 3 Influence of the intrinsic decoherence on nonclassical properties of the system

In this section, we study the influence of the intrinsic decoherence on nonclassical properties of the two-mode Raman coupled model. In order to show how the intrinsic decoherence modifies the time evolution of the atomic inversion, we calculate the expectation values of the operator $\sigma_z$. Using the exact solution $\rho(t)$, we find that atomic inversion

$$P(n_1,n_2,t) = \langle n_1,n_2|\mathrm{Tr}_A[\rho(t)]|n_1,n_2\rangle$$
$$= |Q_{n_1}Q_{n_2}|^2\left\{1-\frac{g^2(n_1+1)n_2}{2\Omega^2(n_1+1,n_2)}\left(1-\cos[2\Omega(n_1+1,n_2)t]\exp\left[-\frac{2t}{\gamma}\Omega^2(n_1+1,n_2)\right]\right)\right\}$$
$$+|Q_{n_1-1}Q_{n_2+1}|^2\frac{g^2n_1(n_2+1)}{2\Omega^2(n_1,n_2+1)}\left\{1-\cos[2\Omega(n_1,n_2+1)t]\exp\left[-\frac{2t}{\gamma}\Omega^2(n_1,n_2+1)\right]\right\} \quad (26)$$

is given by

$$\langle\sigma_z(t)\rangle = \mathrm{Tr}[\rho(t)\sigma_z]$$
$$= 1-g^2\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}|Q_{n_1}Q_{n_2}|^2\frac{(n_1+1)n_2}{\Omega^2(n_1+1,n_2)}$$
$$\times\left\{1-\cos[2\Omega(n_1+1,n_2)t]\exp\left[-\frac{2t}{\gamma}\Omega^2(n_1+1,n_2)\right]\right\}. \quad (25)$$

The probability of finding $n_i$ $(i=1,2)$ photon in the $i$th mode of the radiation field is also found to be

*see equation (26) above.*

It is obvious that both equations (25, 26) in the limit $\gamma \to \infty$ reduce to the usual expression for the atomic inversion and the photon number distribution in the two-mode Raman coupled model governed by the Schrödinger dynamics [6]. It is generally accepted that the revivals of the atomic inversion as well as the oscillations in the photon number distribution appears as a consequence of quantum coherence which are built up during the interaction between the radiation field and atom. From equations (23, 24), we can see that in the time evolution the additional term in the Milburn equation leads to the appearance of the decay factors $\exp[-(2t/\gamma)\Omega^2(n_1+1,n_2)]$ which are responsible for the destruction of the revivals of the atomic inversion and oscillation in the photon number distribution. The numerical result for three values of the decoherence parameter $\gamma$ and two values of the detuning parameter $\Delta$ are shown in Figures 1 and 2 for the time evolution of the atomic inversion (we have set $g=1$). From these figures, we see that for large values of the $\gamma$, the atom exhibits the revival as predicted by the Shrödinger equation. With the decreases of the parameter $\gamma$, we can observe rapid deterioration of the revivals of the atomic inversion. These figures clearly describe the effect of the intrinsic decoherence.

We now turn to discuss the influence of intrinsic decoherence on squeezing of radiation field. We introduce two slowly varying quadrature operators [6]

$$X_1^{(i)}(t) = \frac{1}{2}(a_i\mathrm{e}^{\mathrm{i}\omega_i t}+a_i^\dagger\mathrm{e}^{-\mathrm{i}\omega_i t});$$
$$X_2^{(i)}(t) = -\frac{\mathrm{i}}{2}(a_i\mathrm{e}^{\mathrm{i}\omega_i t}-a_i^\dagger\mathrm{e}^{-\mathrm{i}\omega_i t}). \quad (27)$$

These operators satisfy the commutation relations

$$[X_1^{(i)}(t),X_2^{(j)}(t)] = \frac{\mathrm{i}}{2}\delta_{ij}, \quad (28)$$

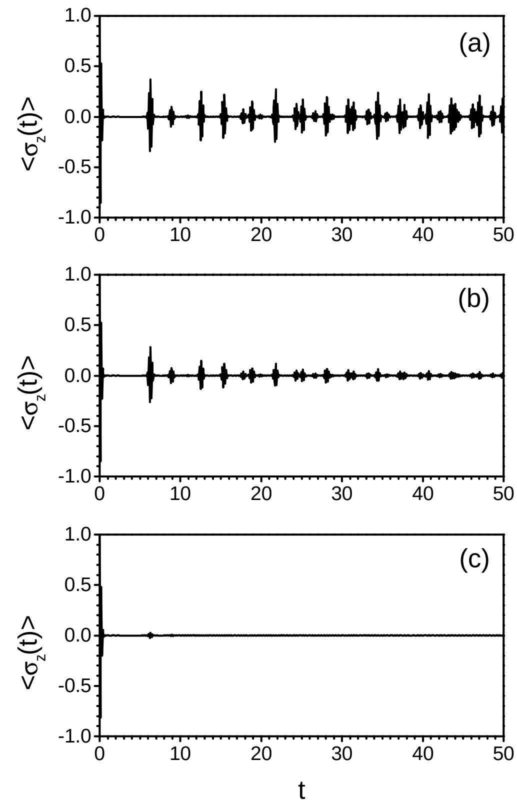


**Fig. 1.** The atomic inversion $\langle\sigma_z(t)\rangle$ as a function of $t$ with $\Delta=0$ for $|\alpha_1|^2=|\alpha_2|^2=15$ and (a) $\gamma=10^5$, (b) $\gamma=10^4$, (c) $\gamma=10^3$.

which implies the Heisenberg uncertainly relations

$$\langle(\Delta X_1^{(i)})^2\rangle\langle(\Delta X_2^{(i)})^2\rangle \geq \frac{1}{16}\cdot \quad (29)$$

Squeezing is said to exist whenever $\langle(\Delta X_j^{(i)})^2\rangle \leq 1/4$ $(i,j=1,2)$.

In order to characterize the influence of intrinsic decoherence on the squeezing, we calculate the $Q$ parameter defined by

$$Q_j^{(i)} = \frac{\langle(\Delta X_j^{(i)})^2\rangle-0.25}{0.25}, \quad (30)$$

where $-1 \leq Q_j^{(i)} < 0$ for squeezing.

$$\langle N_1\rangle = |\alpha_1|^2 + \frac{1}{2}(1 - \langle\sigma_z(t)\rangle); \tag{32}$$

$$\begin{aligned}\langle a_1^2\rangle &= \frac{1}{4}\sum_{m_1,m_2} Q^*_{m_1} Q_{m_1+2}|Q_{m_2}|^2\\
&\times\{\exp(\mathrm{i}r_+t - \frac{t}{2\gamma}r_+^2)[\sqrt{(m_1+1)(m_1+2)}(1+\frac{\Delta}{2\Omega(m_1+1,m_2)})(1+\frac{\Delta}{2\Omega(m_1+3,m_2)}) + \frac{g^2\sqrt{(m_1+1)(m_1+2)}(m_1+3)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+3,m_2)}]\\
&+\exp(\mathrm{i}r_-t - \frac{t}{2\gamma}r_-^2)[\sqrt{(m_1+1)(m_1+2)}(1-\frac{\Delta}{2\Omega(m_1+1,m_2)})(1-\frac{\Delta}{2\Omega(m_1+3,m_2)}) + \frac{g^2\sqrt{(m_1+1)(m_1+2)}(m_1+3)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+3,m_2)}]\\
&+\exp(\mathrm{i}s_+t - \frac{t}{2\gamma}s_+^2)[\sqrt{(m_1+1)(m_1+2)}(1+\frac{\Delta}{2\Omega(m_1+1,m_2)})(1-\frac{\Delta}{2\Omega(m_1+3,m_2)}) - \frac{g^2\sqrt{(m_1+1)(m_1+2)}(m_1+3)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+3,m_2)}]\\
&+\exp(\mathrm{i}s_-t - \frac{t}{2\gamma}s_-^2)[\sqrt{(m_1+1)(m_1+2)}(1-\frac{\Delta}{2\Omega(m_1+1,m_2)})(1+\frac{\Delta}{2\Omega(m_1+3,m_2)}) - \frac{g^2\sqrt{(m_1+1)(m_1+2)}(m_1+3)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+3,m_2)}]\};\end{aligned} \tag{33}$$

$$\begin{aligned}\langle a_1\rangle &= \frac{1}{4}\sum_{m_1,m_2} Q^*_{m_1} Q_{m_1+1}|Q_{m_2}|^2\\
&\times\{\exp(\mathrm{i}a_+t - \frac{t}{2\gamma}a_+^2)[\sqrt{m_1+1}(1+\frac{\Delta}{2\Omega(m_1+1,m_2)})(1+\frac{\Delta}{2\Omega(m_1+2,m_2)}) + \frac{g^2\sqrt{m_1+1}(m_1+2)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+2,m_2)}]\\
&+\exp(\mathrm{i}a_-t - \frac{t}{2\gamma}a_-^2)[\sqrt{m_1+1}(1-\frac{\Delta}{2\Omega(m_1+1,m_2)})(1-\frac{\Delta}{2\Omega(m_1+2,m_2)}) + \frac{g^2\sqrt{m_1+1}(m_1+2)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+2,m_2)}]\\
&+\exp(\mathrm{i}b_+t - \frac{t}{2\gamma}b_+^2)[\sqrt{m_1+1}(1+\frac{\Delta}{2\Omega(m_1+1,m_2)})(1-\frac{\Delta}{2\Omega(m_1+2,m_2)}) - \frac{g^2\sqrt{m_1+1}(m_1+2)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+2,m_2)}]\\
&+\exp(\mathrm{i}b_-t - \frac{t}{2\gamma}b_-^2)[\sqrt{m_1+1}(1-\frac{\Delta}{2\Omega(m_1+1,m_2)})(1+\frac{\Delta}{2\Omega(m_1+2,m_2)}) - \frac{g^2\sqrt{m_1+1}(m_1+2)m_2}{\Omega(m_1+1,m_2)\Omega(m_1+2,m_2)}]\},\end{aligned} \tag{34}$$

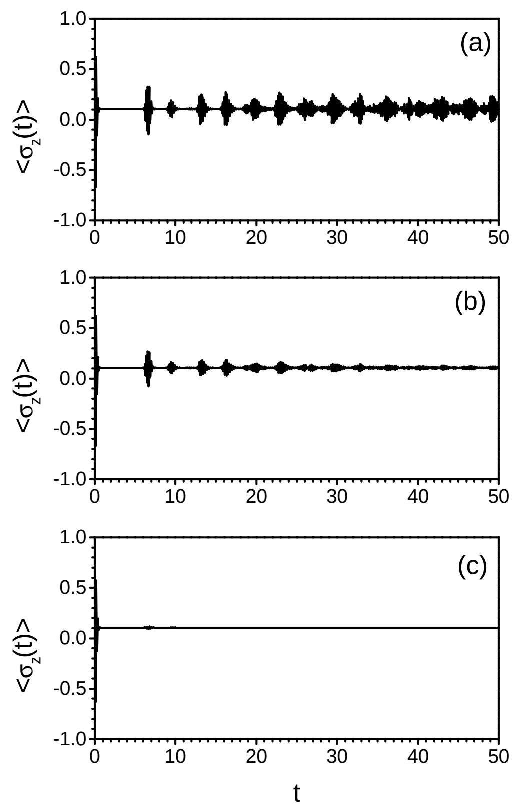


**Fig. 2.** The atomic inversion $\langle\sigma_z(t)\rangle$ as a function of $t$ with $\Delta = 10$ for (a) $\gamma = 10^5$, (b) $\gamma = 10^4$, (c) $\gamma = 10^3$. All other conditions are the same as in Figure 1.

Through some algebraic manipulations, we give the expression of $Q_j^{(1)}$ for the two-mode Raman coupled model

$$\begin{aligned}Q_1^{(1)} &= 2\langle N_1\rangle + \langle a_1^2\rangle e^{2\mathrm{i}\omega_i t} + \langle a_1^{\dagger 2}\rangle e^{-2\mathrm{i}\omega_i t}\\
&\quad - [\langle a_1\rangle e^{\mathrm{i}\omega_i t} + \langle a_1^\dagger\rangle e^{-\mathrm{i}\omega_i t}]^2;\\
Q_2^{(1)} &= 2\langle N_1\rangle - \langle a_1^2\rangle e^{2\mathrm{i}\omega_i t} - \langle a_1^{\dagger 2}\rangle e^{-2\mathrm{i}\omega_i t}\\
&\quad + [\langle a_1\rangle e^{\mathrm{i}\omega_i t} - \langle a_1^\dagger\rangle e^{-\mathrm{i}\omega_i t}]^2,\end{aligned} \tag{31}$$

where

*see equations (32–34) above.*

with

$$\begin{aligned}r_\pm &= -2\omega_1 \pm \Omega(m_1+1,m_2) \mp \Omega(m_1+3,m_2);\\
s_\pm &= -2\omega_1 \pm \Omega(m_1+1,m_2) \pm \Omega(m_1+3,m_2);\\
a_\pm &= -\omega_1 \pm \Omega(m_1+1,m_2) \mp \Omega(m_1+2,m_2);\\
b_\pm &= -\omega_1 \pm \Omega(m_1+1,m_2) \pm \Omega(m_1+2,m_2).\end{aligned} \tag{35}$$

From equations (31–35), we can see that in the time evolution the additional term in the Milburn equation leads to the appearance of the decay factors in each term in the expression of the $Q$ parameter. In Figure 3, we plot the parameter $Q_1^{(1)}$ for three values of the decoherence

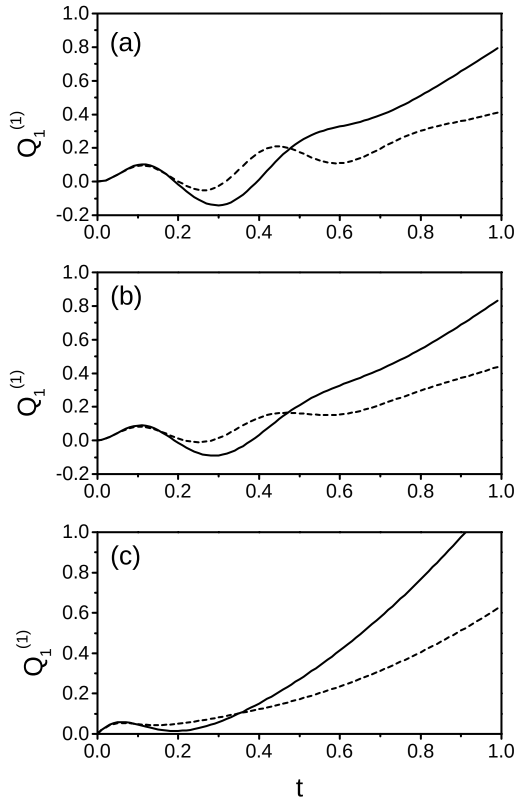


**Fig. 3.** $Q_1^{(1)}$ as a function of $t$ with $\Delta = 0$ (solid line) and $\Delta = 10$ (dashed line) for $|\alpha_1|^2 = 5; |\alpha_2|^2 = 10$ and (a) $\gamma = 10^6$, (b) $\gamma = 10^2$, (c) $\gamma = 10$.

parameter $\gamma$ and two values of the detuning parameter $\Delta$. It is quite clear from these figures that the quadrature squeezing decay with the decrease of the decoherence parameter $\gamma$. The cases of the off-resonant (dashed line) and resonant (solid line) are compared in Figure 3. The off-resonant model is more susceptible to decoherence than resonant model. These figures exhibit the clear influence of the intrinsic decoherence on the squeezing of the radiation field.

## 4 Concluding remarks

We have found the exact solution of the Milburn equation (2) for the two-mode Raman coupled model. Using the exact solution, we have discussed the influence of the intrinsic decoherence on the nonclassical properties of the system such as the revivals of the atomic inversion, oscillation of the photon number distribution and squeezing of the radiation field. It is shown that the intrinsic decoherence in the atom-field interaction suppress the nonclassical effects in the two mode Raman coupled model. The approach adopted here can be extended to the cases of time-dependent atom-field coupling or a $N$-level atom interacting with quantized field modes.

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## References

1. E.T. Jaynes, F. Cummings, Proc. IEEE **51**, 89 (1963).
2. J.H. Eberly, J.J. Sanchez-Mondragon, N.B. Narozhny, Phys. Rev. Lett. **44**, 1323 (1980); G. Rempe, H. Walther, N. Klein, Phys. Rev. Lett. **58**, 353 (1987).
3. H.I. Yoo, J.H. Eberly, Phys. Rep. **118**, 239 (1985).
4. J.R. Kuklinski, J. Madajczyk, Phys. Rev. A **37**, 3175 (1988); G. Rempe, F. Schmidt-Kaler, H. Walther, Phys. Rev. Lett. **67**, 2783 (1990).
5. M. Hillery, Phys. Rev. A **35**, 4186 (1987); P.L. Knight, Tran Quang, Phys. Rev. A **41**, 6255 (1990).
6. C.C. Gerry, J.H. Eberly, Phys. Rev. A **42**, 6805 (1990); D.A. Cardimana, V. Kovanis, M.P. Sharma, A. Gavrielides, Phys. Rev. A **43**, (1991).
7. M. Alexanian, S.K. Bose, Phys. Rev. A **52**, 2218 (1995).
8. Jia-Ren Liu, Yu-Zhu Wang, Phys. Rev. A **54**, 2444 (1996).
9. P.W. Shor, Phys. Rev. A **52**, R2493 (1995); I.L. Chuang, Y. Yamamoto, Phys. Rev. A **55**, 114 (1997).
10. G.J. Milburn, Phys. Rev. A **44**, 5401 (1991).
11. H. Moya-Cessa, V. Buzek, M.S. Kim, P.L. Knight, Phys. Rev. A **48**, 3900 (1993).
12. L.M. Kuang, X. Chen, G.H. Chen, M.L. Ge, Phys. Rev. A **56**, 3139 (1997).